Inexpressible Propositions and Limits of Knowledge, Belief and Assertion

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Overview

- I demonstrate that

(i) The *limitation of logical space and propositions explicated in terms of possible worlds* (entailed by Cantor's theorem and even the theory of recursive/computable functions) cause …

(ii) *limits of the expressive power of language* and thus *also* certain important *propositional notions such as assertion, beleif and knowledge*.

- A naïve approach to the limitations (i) and (ii) leads to a group of famous *paradoxes*, e.g. the Liar Paradox and Kaplan's paradox about possible worlds.

Following Tarski's well-known procedure 1936/1956, paradoxes are best treated as a mere symptom leading to the *revision of our naïve approach to formal(ized) language and its possible model-theoretic interpretation*.

Related work

- A number of results partly supporting our main idea, but often misleadingly framed as 'Liar bussiness':
  - Tichý 1988, Anderson 2009: proof of inexpressibility of a Liar sentence/proposition for Epimenides
    - see also Prior 1961 who first observed that Epimenides cannot express that he is a Liar
      and reactions by Thomason 1986, Asher 1990, Priest 1991, Kaplan 1995, Tucker and Thomason 2011, Bacon with Hawthorne and Uzquiano 2016, Tucker 2018, Bacon with Uzquiano 2018.
  - Tichý 1988: proof of limitation of the assertion relation (with Liar example)
- Two general proofs, but in abstract setting , stated for intepretation of *formal models of modal languages*, i.e. with *possible worlds*; interpretation left to readers:
  - Wiśniewski 2011: proof of existence of inexpressible propositions
  - Fritz 2018: proof of inexpressible propositions, relations etc.

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Possible worlds, PWS-propositions, possible world semantics

Possible world W

A *possible world W* is a logically primitive entity that is a surrogate of a realizable maximal consistent set of state-of-affairs.

Logical space ω

A *logical space ω* is a (at least a two-membered) set of Ws. [note the ω]

- Modal/epistemic logics and type-theoretic approaches to natural language use *possible world semantics* (*PWS*) that deploys possible world *intensions*, esp. *propositions*:

PWS-proposition P

A *PWS-proposition P* is a set of Ws, i.e. a characteristic total or partial ('gappy') function from Ws to truth values, $\omega \to \mathbb{B}$,
where $\mathtt{T}$ – Truth, $\mathtt{F}$ – False; $\mathbb{B} = \{\mathtt{T}, \mathtt{F}\}$.

Cantor's theorem; Theorem about Limitation of Logical Space I

Cantor's theorem

For any set x, $|\mathcal{P}(x)| > |x|$, where $\mathcal{P}$ – powerset, $|\cdot|$ – cardinality.

- Let $\mathcal{P}(\omega)$ be the set of all PWS-propositions over ω.

Corollary

There are more PWS-propositions than possible worlds, $|\mathcal{P}(\omega)| > |\omega|$.

- The corollary obviously entails a well-known ...

Theorem about limitation of logical space I

Every logical space ω is limited, for there is *no 1-1 correspondence between possible worlds and PWS-propositions*.

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Formal tool

- In what follows, a particular *formal tool* for manipulating PWS-propositions (and other things) is chosen.
- First, its simple *type-theoretic part*, as *higher-order logic* allows *quantification over functions-as-mappings*, incl. PWS-propositions.
  - These functions may lack a value for some argument, being partial, which is quite natural, cf. maths + CS (comp.science) + linguistics.
- Second, the type theory is general even in the sense that *quantification over functions-as-computations* (i.e. *algorithms*) directly manipulates fine-grained meanings of expressions.
  - This assumption may be lifted if one identifies expressions with their meanings (and pays the price).
- Third, *typing* of expressions is quite natural, cf. not only CS and linguistics.
  - Natural linguistic typing (V, VP, N, NP, etc...) and even Tarski's levels of T-predicate match are examples of typing.

Sentences need fine-grained hyperintensional meaning

- The need for *hyperintensional fine-grained meanings* (e.g. Lewis 1970, Cresswell 1975, 1983):
  - a. *hyperintensional*: to differentiate e.g. between $1 + 2 = 3$ and Fermat's Last Theorem and to avoid the *Logical Omniscience Problem*;
  - b. *fine-grained*: to capture the difference in semantic content of e.g. $1 + 2 = 3$ and $2 + 1 = 3$.
- The desiderata a. and b. are fulfilled by *Neo-Fregean algorithmic conceptions* (Tichý 1988, Moschovakis 1994); they go beyond PWS:

Neo-fregean algorithmic meaning

An expression E expresses an algorithmic *meaning*, called by Tichý *construction*, which *determines* E's *denotatum* (if any).

- Tichý's constructions satisfy *'intensional' criterion of individuation*: they can be equivalent without being identical.

Type theory TT*

- *Simple type theory* (*STT*) (esp. Church 1941, Coquand 2015, ...) with rules for logical operators provides *higher-order logic* (⟶ great expressivity).

- *Ramified TTs* (*RTTs*) incorporate *typing*: intensional operators T or K are 'stratified' into hierarchies that consist of *orders* ('levels').
  a. known for solving a number of *semantic and epistemic paradoxes* (e.g. Russell 1908, Church 1976)
  b. studied even within CS (e.g. Kamareddine, Laan and Nederpelt 2010)

- I will deploy a heavy modification of Tichý's 1982–1988 TT, called *TT** (Raclavský 2018+). It ...
  a. includes STT;
  b. its ramified part enables *non-circular individuation* of constructions;
  c. TT* embraces of *partiality* of functions and partial term evaluation.

v-improperness, v-congruence, the language $\mathcal{L}_{TT*}$

- Let $[\![C]\!]^{\mathscr{M},v}$ be read as C *v-constructs* (in $\mathscr{M}$), where v is an *assignment*.

v-Improperness; v-Congruence

C is *v-improper* iff there is no x s.t. $x = [\![C]\!]^{\mathscr{M},v}$ (partiality/abortivity).

C_1 is *v-congruent* ($\cong$) with C_2 iff $[\![C_1]\!]^{\mathscr{M},v} = [\![C_2]\!]^{\mathscr{M},v}$, or C_1 and C_2 are both v-improper.

Language $\mathcal{L}_{TT*}$ (schematic)

$C^\tau :=$

$x^\tau \; |\mathbf{X}^\tau \mid C_0^{\tau_0}(C_1^{\tau_1}, ..., C_m^{\tau_m}) \mid \lambda x_1^{\tau_1} ... x_m^{\tau_m}.C^\tau \mid \ulcorner C^\tau \urcorner$ (acquisitions).

for any *type* τ, any *construction* C, any construction of a non-construction X.

  - τ is often dropped (or simplified), [and] are auxiliary brackets
  - Typing statements C/τ read "C should v-determine a τ-object".

Types, orders, models, frames for TT*

Types and orders for TT*

	Type hierarchy TT$^+$	*extensions of type language*
i.	*base types*	$\tau_1^B, \ldots, \tau_m^B$
ii.	*1st-order types* τ^1	$\tau^1 ::= \ \tau^B \mid \langle \tau_1^1, \ldots, \tau_m^1 \rangle \rightarrow \tau^1$
iii.	*types of nth-order constructions*	$*^n$
iv.	*$(n+1)$st-order types* t^{n+1}	$\tau^{n+1} ::= \ *^n \mid \tau^n \mid \langle \tau_1^{n+1}, \ldots, \tau_m^{n+1} \rangle \rightarrow \tau^{n+1}$

- *Intepretation of types*: $\mathscr{D}_{\tau_i^B}$ is a set of all primitive objects of a given ‘kind’; $\mathscr{D}_{\langle \tau_1^n, \ldots \tau_m^n \rangle \rightarrow \tau^n}$ is a set of all functions from $\mathscr{D}_{\tau_1^n} \times \ldots \times \mathscr{D}_{\tau_m^n}$ to $\mathscr{D}_{\tau^n}$.

Frames and models for TT*

A *model* $\mathscr{M}$ is $\langle \mathscr{F}, \mathscr{I} \rangle$ where $\mathscr{F}$ is a *frame* $\{\mathscr{D}_{\tau_i}\}_{i \in \mathbb{N}}$ and $\mathscr{I}$ is an *interpretation function* from constants of $\mathcal{L}_{TT*}$ to members of $\mathscr{F}$.

Denotational semantics for terms of $\mathcal{L}_{TT*}$, i.e. v-determining

- Let $\tau_{(i)}$ be any *type* (a set of τ-objects), s^τ a sequence of τ-objects, x be a variable for constructions or non-constructions, X be a *construction of a non-construction*, $C_{(i)}$ be a *construction*, c be a variable for constructions, f be a function (a map), v be a *assign ment*, $\mathscr{M}$ be a *model*, and _ nothing at all (partiality) (Raclavský 2018, 2024)

1. $[\![x_k^\tau]\!]^{\mathscr{M},v}$ = the only x such that $s^\tau \in v$ and $x = s^\tau(k)$
2. $[\![\mathbf{X}]\!]^{\mathscr{M},v} = \mathcal{I}(X)$
3. $[\![\ulcorner C \urcorner]\!]^{\mathscr{M},v} = C$
4. $$[\![\, C(C_1, \ldots, C_m) \,]\!]^{\mathscr{M},v} = \begin{cases} f(X_1, \ldots, X_m) \text{ if } [\![C]\!]^{\mathscr{M},v} = f \in \langle \tau_1 \ldots \tau_m \rangle \rightarrow \tau, \\ \qquad [\![C_1]\!]^{\mathscr{M},v} = X_1 \in \tau_1, \ldots, \text{ and } [\![C_m]\!]^{\mathscr{M},v} = X_m \in \tau_m \\ \_\text{(undefined; partiality)} \end{cases}$$
5. $[\![\, \lambda x_1^{\tau_1} \ldots x_m^{\tau_m} . C \,]\!]^{\mathscr{M},v}$ = the only $f \in \langle \tau_1 \ldots \tau_m \rangle \rightarrow \tau$ which maps each $\langle [\![x_1^{\tau_1}]\!]^{\mathscr{M},v_{(i)}}, \ldots, [\![x_m^{\tau_m}]\!]^{\mathscr{M},v_{(i)}} \rangle$ to $[\![C]\!]^{\mathscr{M},v_{(i)}}$ (if any), where v_i is like v except what it assigns to other variables than $x_1^{\tau_1}, \ldots, x_m^{\tau_m}$, $v_1 \neq \ldots \neq v_m$ and $[\![x_{(i)}^{\tau_i}]\!]^{\mathscr{M},v_{(i)}} \in \tau_{(i)}$

Natural deduction in sequent style for TT*

- *Natural deduction in sequent-calculus style for TT** (*ND*$_{TT*}$) gives a *proof-theoretic specification of constructions*.
  - Kuchyňka and Raclavský 2024 as a (large) modification of Tichý 1982; see Henkin-completeness proof for ND$_{TT*}$.

i. basic items are *matches* $\mathcal{M}$ of the for $C \cong x$ and $C \cong X$

- it is needed e.g. form managing v-improper constructions

ii. *sequents* $\mathcal{S}$ are of the form $\Gamma \Rightarrow \mathcal{M}$, where Γ is a set of matches

iii. *derivation rules* $\mathcal{R}$ are of the form

$$\frac{\mathcal{S}_1; \ldots; \mathcal{S}_m}{\mathcal{S}}$$

- *Quantifiers* $\forall^\tau, \exists^\tau / (\tau \rightarrow o) \rightarrow o$ behave as in *strong Kleene logic*, *connectives* $\neg / o \rightarrow o$; $\wedge, \vee, \rightarrow / (o \rightarrow o) \rightarrow o$ behave as in *weak Kleene*.
- *Definitions* are bi-directional rules; often written as $\models C_1 \Leftrightarrow C_2$.

Main types of extensions/intensions of an applied TT*, THL

Types τ^B	*entities in* τ^B	*vars ranging over* τ^B
base types		
ι	*individuals*	$x, y, \ldots$
o	*truth values*	$o, o', \ldots$
ω	*possible worlds*	$w, w', \ldots$
ρ	*real numbers/time instants*	– suppressed bellow
types t *of main*	*extensions*	
$\tau \rightarrow o$	*sets* of τ-objects (i.e. objects of type τ)	
$\langle \tau_1, \ldots, \tau_m \rangle \rightarrow o$	*relations* (-in-extension) among $\tau_1, \ldots, \tau_m$-objects	
types of main	(PWS-) *intensions*	
π	*propositions*	$p, q, \ldots$
$\tau \rightarrow \pi$	*properties* of τ-objects	
$\langle \tau_1, \ldots, \tau_m \rangle \rightarrow \pi$	*relations* (-in-intension) among $\tau_1, \ldots, \tau_m$-objects	

where π is short for $\omega \rightarrow o$

Meanings explicated in TT*/THL

Constructions of TT* as meanings

Meanings are explicated as constructions. (Tichý 1988)
Sentential meanings, *'propositions'* are explicated as constructions of truth values, called **-propositions*. (Kuchyňka, Raclavský 2019+)

Examples	*which is short for*
$3 \div 0 =^{\rho} 0$	$=^{\rho} (\div(3,0),0)$ (no truth value)
$\mathbf{Dog}(\mathbf{Fido})_w$	$[\mathbf{Dog}(\mathbf{Fido})](w)$
$\forall x[\mathbf{Dog}(x)_w \wedge 3 \div 0 =^{\rho} 0]$	$\forall^{\iota}(\lambda x^{\iota}[\mathbf{Dog}(x)_w \wedge 3 \div 0 =^{\rho} 0])$
$\mathbf{Believe}_{\mathbf{Ann}}(\ulcorner\mathbf{Dog}(\mathbf{Fido})_w\urcorner)_w$	$[\mathbf{Believe}(\mathbf{Ann}, \ulcorner\mathbf{Dog}(\mathbf{Fido})_w\urcorner)]_{(w)}$

Belief/propositional attitudes are relations-in-intension between agents and *-propositions C delivered e.g. by the acquisitions $\ulcorner C\urcorner$.

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Plan of this section

- All belief/propostional attitudes are of type $\langle \iota, *^n \rangle \rightarrow \pi$.

I am going to show that

- *believing* Bel and *assertion* As are explicable as certain *relations (-in-intension) between individuals and *-propositions*.
- But certain ⟨individual X, *-proposition C ⟩ couples are logically excluded from any possible extension of these relations,
  - In other words, *explication of* Bel *and* As *is limited*.
  - The limitation follows from the limitation of logical space.

- Assertion-relation will be studied on the particular example 'asserting-C-that-is-not-true' ($\sim$ 'being a liar about C').
  - a close variant: 'asserting-C-that-is-not-meaningful' ($\sim$ the Paradox of Non-Communicator, 1960s).

‘Propositional’ notions and their explication

‘Propositional’ notions

‘Propositional’ notions (*P-notions*) are *modal notions* applicable to *-propositions,
e.g. knowing K, necessity $\square$, truth Tr, believing Bel, assertion As

K is explicated by (a hierarchy of) construction(s) $\mathbf{K}^n / \langle \iota, *^n \rangle \to \pi$
determining a *relation between individuals and nth-order *-propositions*.
Similarly for: (a) *dyadic* P-notions $\mathbf{Bel}^n, \mathbf{As}^n / \langle \iota, *^n \rangle \to \pi$
(b) *monadic* P-notions $\mathbf{Tr}^n, \square^n / *^n \to \pi$

- *Examples* of rules governing P-notions:
  - only $K, \square, Tr$ are governed by the *T-axiom/T-rule*, cf. e.g. the *Rule of Factivity of K* let $c^n / *^n$; $\mathbf{\Gamma}^o / *^n \to o$

$$\frac{\mathbf{K}^n_x(c^n)_w \cong \mathtt{T}}{\mathbf{\Gamma}^o(c^n)_w \cong \mathtt{T}}$$

$$\models \square^n(c^n)_w \cong o \Leftrightarrow \forall w[\mathbf{\Gamma}^o(c^n)_w =^o \mathtt{T}] \cong o$$

$$\models \mathbf{Tr}^n(c^n)_w \cong o \Leftrightarrow \exists o[\Gamma^o(c^n)_w =^o o \wedge o =^o \mathtt{T}] \cong o$$

Theorem about limitation of logical space II

Theorem about limitation of logical space II

For every logical space ω and at least a two-membered domain of individuals ι it holds that there is *no 1-1 correspondence between PWS-propositions and binary relations (-in-intension)*.

- clearly, $|\, Ws \,| < |\, Ps \,| < |\, Rs \,|$, where R is a *relation-in-intension*

- *Proof* (sketch): even if we considered sets ω and ι, each of cardinality 2, and admitted only total propositions, there would only be 4 propositions while there would be much more properties of type $\iota{\rightarrow}\pi$ (namely 16), and even more relations e.g. of type $\langle\iota,\pi\rangle{\rightarrow}\pi$

(Meta)theorem about limitation of logical space III

- Given the notion of construction of objects: to each member of a set of v-congruent *-propositions there corresponds a *corresponding PWS-proposition*:

A PWS-proposition corresponding to *-propositions

$$\models \mathbf{Corresp}^n(p, c^n) \Leftrightarrow [p =^\pi \lambda w.\Gamma^o(c^n)_w]$$

- To proceed from the level of PWS-propositions to the level of *-propositions, we may use

(Meta)theorem about limitation of logical space III

The limitation of logical space mentioned in *theorems of type I and II is replicated on the level of *-propositions*
due to the link between *-propositions and propositions corresponding to them.

Limitation of explication of 'propositional' notions such as Bel and As

Theorem about limitation of explication of 'propositional' notions such as Bel and As

Some 'propositional' notions, e.g. As and Bel,
which *are not governed by the T-axiom*,

cannot be explicated so that

> an agent X could bear the relation of assertion/belief towards a *-proposition C^1 such that

i. C^1 is v-congruent with the *-proposition D (of order $n > 1$) and

ii. D says that X asserts/believes C^1, while C^1 is not true.

- Next 3 slides provide a *proof* of the theorem for the case of As
  - a proof for the case of Bel is similar

Proof 1/3: the General Principle of 'Reducibility' (GPR)

- In a full-fledged ramified TT (argued for by Church 1976), a certain 'reducibility' principle holds and is expressible as a derivation rule
- In TT*, there is its rather general form

General Principle of Reducibility (GPR)

For any $(n+1)$st-order *-proposition C^{n+1} there exists an nth-order v-congruent *-proposition C^n, for $1 < n$.

  - It's obviously incorporated within the vast expressibility of $\mathcal{L}_{TT*}$ (Raclavský 2009, 2019)

- Linguistic analogy: to any (compound) expression E^n of a 'level' n, there is its 1-'level' equivalent mate, a *simple* expression E^1 (a constant); the equivalency is provided by $E^1 \Leftrightarrow E^n$.

Proof 2/3: The Liar Paradox and the General Reducibility Principle

- The GPR *restores* the Liar Paradox (already Tichý 1988, Giaretta 1998), it's *Revenge Liar* for Typing Solution.

To be (1st-order) liar

$$\models \mathbf{Li}^1(x)_w \Leftrightarrow \exists c^1[\mathbf{As}^1_x(c^1)_w \wedge \neg\mathbf{Tr}^1(c^1)_w]$$

where $\mathbf{Li}$ (to be a 1st-order liar)$/\iota{\rightarrow}\pi$

  - Unlike the *2nd-order *-proposition* $\exists c^1[\mathbf{As}^1_x(c^1)_w \wedge \neg\mathbf{Tr}^1(c^1)_w]$: $\mathbf{Li}^1(x)$ is a *1st-order *-proposition* and so it occurs in the range of c^1 (typing cannot prevents that), so the Liar strikes back (cf. the note on the Liar below).

- Note: the definition is of the form $C^1 \Leftrightarrow D$, where *As is the relation R whose explication is limited*
  - Now we'll use the *Principle of Non-contradiction* (*PNC*) to show that D can never be easily paired with C^1 (from Tichý 1988):

Proof 3/3: X cannot assert that X is a liar

- Let $C^1 := \mathbf{Li}(\mathbf{X})_w$ and $D := \mathbf{As}^1_{\mathbf{X}}(C^1)_w \wedge \neg\mathbf{Tr}^1(C^1)_w$ where $\mathbf{X}/\iota$
- Assume $\omega = \{W_1, W_2\}$; then, every *-proposition is of just one ‘kind’ E_i, for $1 \leqslant i \leqslant 9$

	E_1	E_2	E_3	E_4	E_5	E_6	E_7	E_8	E_9
W_1	T	T	T	F	F	F	_	_	_
W_2	T	F	_	T	F	_	T	F	_

- The following demonstrates that *D is never υ-congruent with C^1*:

If W is ...	*and X is related by R to C^1 of kind ...*	*then D is of kind ...*
W_1	E_1, E_2, or E_3	E_4, E_5, or E_6
	E_4, E_5, E_6, E_7, E_8, or E_9	E_1, E_2, or E_3
W_2	E_1, E_4, or E_7	E_2, E_5, or E_8
	E_2, E_3, E_5, E_6, E_8, or E_9	E_1, E_4, or E_7

- Similar proof for $R :=$ *believe* (and ‘*be a fool*’ instead of ‘*be a liar*’)

A note on the Liar paradox and utterance

- The theorem thus brings a PNC-based *solution* to the Liar paradox:
  - because of the reducibility principle, TT* cannot solve the Liar by mere 'typing' (as it does solve the paradox with "The sentence is not true in L^1")
- Since the assumption that X asserts, uttering "X is a liar", that she is a liar leads to contradiction ...

We must maintain that *by uttering "X is a liar", X cannot bring about the world being such that X truly asserts that she is a liar*

a. X's utterance is 'abortive'

b. preventing criticism: the sentence "X is a liar" *still has its conventional meaning*, it's deployable by people other than X

- A partial conclusion:

'Inexpressible proposition' P may only mean that, *in some context of utterance, P is not assertible using an expression for an agent.*

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I am going show that

- *knowing K* is explicable as a certain *relation (-in-intension) between individuals and *-propositions*.
- But certain ⟨individual X, *-proposition C^n⟩ couples *are logically excluded* from any possible extension of the relations.
- In other words, *explication of K is limited*,
  while the limitation follows from the limitation of logical space.

- The C^n in question is of a special sort; it will be denoted by $\boxed{K}$.
- Not enough space to show that the so-called $\boxed{P}$-problem also affects Bel and As, but not Tr and $\square$.

Limitation of explication of ‘propositional’ notions such as K

Theorem about limitation of explication of ‘propositional’ notions such as K

Some ‘propositional’ notions, e.g. K,

which *are governed by the T-axiom*,

cannot be explicated so that

> an agent X could bear the relation of knowing towards
> a *-proposition C^1 such that

i. C^1 is v-congruent with the *-proposition D (of order $n > 1$) and
ii. D says that X does not know C^1.

- For proof, we cannot use the same procedure as in the previous section, because K is factive:

  $D := \mathbf{K}^1_x(c^1)_w \wedge \neg\mathbf{Tr}^1(c^1)_w$ is self-contradictory

  – instead, we'll use *Self-referential Lemma*

Self-referential Lemma and the *-proposition $\boxed{K^1}$

- The *Self-referential (Diagonal) Lemma* was proved (by Carnap) for formulas and predicate symbols, but it obviously holds even for certain *-propositions:

Self-referential Lemma

$\vdash \Gamma^{\tau}(c^n) \Leftrightarrow \neg C^n(c^n)$ where $C^n / *^n \rightarrow o; c^n / *^n$

  - without the GPR, the formulation of the lemma would not be possible

- An instance of the lemma:

***-proposition $\boxed{K^1}$**

$\models \boxed{K^1} \Leftrightarrow \neg \mathbf{K}^1_x(\ulcorner \boxed{K^1} \urcorner)_w$ where $\boxed{K^1}/o$

Impossibility of knowing the *-proposition $\boxed{K^1}$

- Necessarily, $\boxed{K^1}$ cannot by known by any x.
  - where $\underset{o}{\square}/\pi\rightarrow o$ is defined by $\models \underset{o}{\square}(p) \Leftrightarrow \forall w[p_w =^o \mathtt{T}]$
- *proof* is obvious

- That one cannot know the *-proposition that she does not know this *-proposition sounds paradoxical – the *Paradox of Impossibility to Know One's Own Ignorance* (as 'Socrates' paradox' might e called)
- Plain *verificationism* is thus clearly wrong

An agent may seem to attempt to be intentionally related to such $\boxed{K^1}$, but it's impossible for her. It's sort of 'blindspot'.

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Some conclusions 1/2

- *An explication of notions such as* K, Bel, As *is limited because of the limitation of logical space*:

A 'propositional' notion C^n	*subjected to T-axiom*	$\boxed{P}$*-problem*	C^n *v-determines an object of type*
$\mathbf{Tr}^n$	✓	×	$*^n \rightarrow \pi$
$\Box^n$	✓	×	$*^n \rightarrow \pi$
$\mathbf{K}^n$	✓	✓	$\langle \iota, *^n \rangle \rightarrow \pi$
$\mathbf{Bel}^n$	×	✓	$\langle \iota, *^n \rangle \rightarrow \pi$
$\mathbf{As}^n$	×	✓	$\langle \iota, *^n \rangle \rightarrow \pi$

- Some of our intuitive possible worlds are *'impossible possible worlds', not genuine possible worlds.*
  - As pseudo-worlds they shouldn't be used for explication of propositional notions and intensions in general.

Some conclusions – future prospects 2/2

- The Liar paradox and its many mates (Bouleus's paradox – belief, Non-Communicator p. Socrates' p., ...) indicate the expressive limitations of any language and explication of the propositional notions.
- Their unite investigation concerning their (dis)similarities (as pronounced by Fitch 1963) is still in progress.
- As pointed out by Anderson (1983),

> *I think two things will be necessary: epistemic logic should be incroporated into a logic that treats not just knowledge and belief, but also inference, understanding, and perhaps other propositional attitudes, and ties with more central parts of logic must be established —presumably proof theory, and more, generally, the theory of recursive functions. (pp. 338–339)*